Rain Outage Performance of Tandem and Path Diversity 18-GHz Short Hop Radio Systems

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(Manuscript received August 19, 1970)

The performance with respect to rain outage time of dual path diversity and non-path diversity (tandem) arrangements for 18-GHz short hop radio systems is computed and compared. The analysis is based on two extrapolations of R. A. Semplak's[1] *three-year average of the measured probability distributions for rain attenuation at 18.5 GHz on a 6.4-km hop in New Jersey. The effects of merge hops and joint fading between hops in the diversity system, dependence of the rain attenuation distribution on hop length, and uncertainty in the tail of the distribution are included.*

The results show that (i) the performance of tandem systems relative to diversity systems increases as the system length increases, (ii) the difference in the number of repeaters per unit length required for short and long tandem systems is small, (iii) the performance of the diversity system is strongly dependent on the amount of joint fading between parallel paths, and (iv) the performance of the tandem system is strongly dependent on the tail of the attenuation distribution. Neither of the latter two factors is known from rain attenuation measurements, but if the joint attenuation probabilities are sufficiently high, then diversity shows no advantage over tandem for either of the assumed extrapolations. The uncertainty in the tail of the attenuation distribution and the sensitivity of the tandem system performance to it emphasize the need for reliable attenuation measurements out to a probability of about 10^{-7}.

I. INTRODUCTION

The problems of utilizing the frequencies above 10 GHz for radio relay communication have been discussed by L. C. Tillotson[2]; the major technical problem is attenuation by rain. Based on results derived from a rain gauge network in Bedfordshire, England, which

showed that the joint distribution of rain rates for two laterally separated paths is significantly less than the distributions for the individual paths, D. C. Hogg[3] suggested the use of dual path diversity as a means for reducing the magnitude of the problem. However, in deciding the value of path diversity, many other factors must be considered such as variation of fade margin with hop length, merging of the diversity paths at switching points, the nature of the attenuation distributions, and the degree of independence of two laterally separated paths versus separation.

L. T. Gusler[4,5] has previously calculated the reliability and maximum lengths of diversity and nondiversity systems at 11 and 17 GHz based on attenuation distributions derived from six rain gauges on a five-mile path in New Jersey, and assumptions on joint distributions, merge paths, and repeater parameters. He concluded that diversity systems with short hops and moderate fade margins are suitable for long-haul systems, but that nondiversity systems are suitable only for short-haul or low-reliability systems. These results have since been used in studies of long-haul microwave pole line systems.[6]

This paper describes a new comparison of the rain outage performance of 18-GHz dual path diversity (diversity) systems and non-path diversity (tandem) systems including the effects of merge paths in the diversity system, joint fading between diversity paths, dependence of the rain attenuation distribution on hop length, and uncertainty in the tails of the rain attenuation distribution. Characteristics other than rain outage performance, such as path switching, delay equalization of the diversity paths, equipment redundancy, and interference may weigh for or against a particular system, but these factors are not considered here.

II. SUMMARY

The tandem and diversity systems are compared on the basis of equal numbers of identical repeaters in a length which is equal to the length of a diversity switching section; the section geometry is shown in Fig. 1. Lengths of tandem and diversity systems with 0.01 percent total rain outage time are calculated from the rain attenuation distribution as a function of tandem hop length with section length and the repeater fade margin as parameters. Diversity path separation is also a parameter in the diversity case and spacings of 10 and 20 km are used in the computations. System lengths for outage times other than 0.01 percent are inversely proportional to the outage time. To

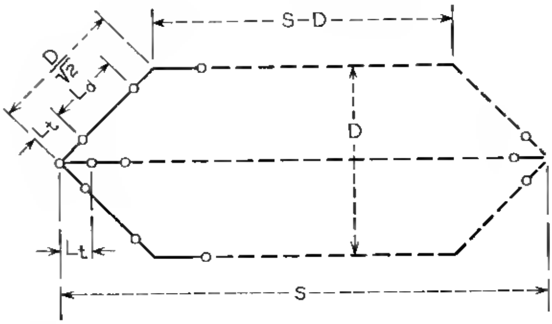

Fig. 1—Geometry of tandem and diversity systems.

approximate the reduced repeater spacing necessary at the merge points in the diversity system, the lengths of tbe first and last hops in each diversity path are set equal to tbe hop length in the comparison tandem system.

The rain outage performance of a tandem system depends on the probability distribution of rain attenuation whicb is a function of the hop length; diversity system performance depends on the joint probability distributions for laterally separated parallel hops. The measured 18-GHz attenuation distribution for a 6.4-km hop in Holmdel, New Jersey, reported by R. A. Semplak[1], is used as the basic distribution. To extrapolate and to include the effects of variations in the tail of the distribution, the measured distribution is approximated by two functions, A and B. The measured distribution for the 6.4-km hop and its approximations are shown in Fig. 2 as curves R1, A1, and B1. A hop length dependence is assumed which causes the approximation to fit the measured 30.9-GHz (1.9-km hop length) distribution reported by Semplak[1] when it is converted to an equivalent 18.5-GHz distribution. This derived 18.5-GHz distribution for a 1.9-km hop and its approximations are shown as curves R2, A2, and B2 in Fig. 2. Figure 3(a) shows the assumed dependence of attenuation on hop length for constant probability and Fig. 3(b) shows the family of distributions obtained from the approximating function B with hop length as a parameter.

Joint fading of the directly opposite and first diagonally opposite hops in the diversity system is included in the following way. Wben two bops are statistically independent, the distribution of the attenuation which is exceeded jointly on both hops is the product of the indi-

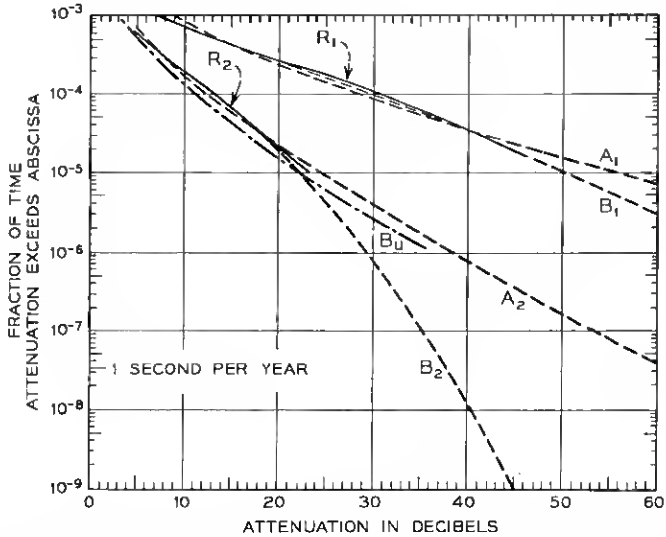

Fig. 2—Probability distributions for rain attenuation at 18 GHz. Curves are: R1, measured at 18.5 GHz on 6.4-km hop (Ref. 1); R2, 18 GHz distribution for 1.9-km hop derived from 30.9-GHz distribution (Ref. 1); A1 and A2, approximating function A for 6.4-km and 1.9-km hop lengths; B1 and B2, approximating function B for 6.4-km and 1.9-km hop lengths; Bu, 1.9-km distribution derived from one minute rain rate distribution (Ref. 13) using equation (1) with $k = 0.1$.

vidual distributions; when the two hops are completely dependent (conditional probabilities are unity) the joint distribution is equal to the individual distribution. Joint distributions lying between these two special cases are assumed to be given by the individual distribution raised to an exponent, c, which has values $c = 2$ in the independent case and $c = 1$ in the completely dependent case. Diversity system performance is calculated for distributions corresponding to c equal to 2.0, 1.8, 1.6, 1.4, and 1.0. These distributions are shown in Fig. 4 for the approximation B1 in Fig. 2. The results of the analysis can be applied, once the measured joint attenuation distribution is known, by finding the corresponding exponent, c, for the measured distribution.

All hops other than the directly opposite hops and the first diagonally opposite hops in the diversity systems are assumed to be statistically independent. However, joint fading of other pairs of hops further decreases the performance of the diversity system relative to the tandem system. Therefore the preceding assumptions establish minimum conditions under which the diversity system must have superior

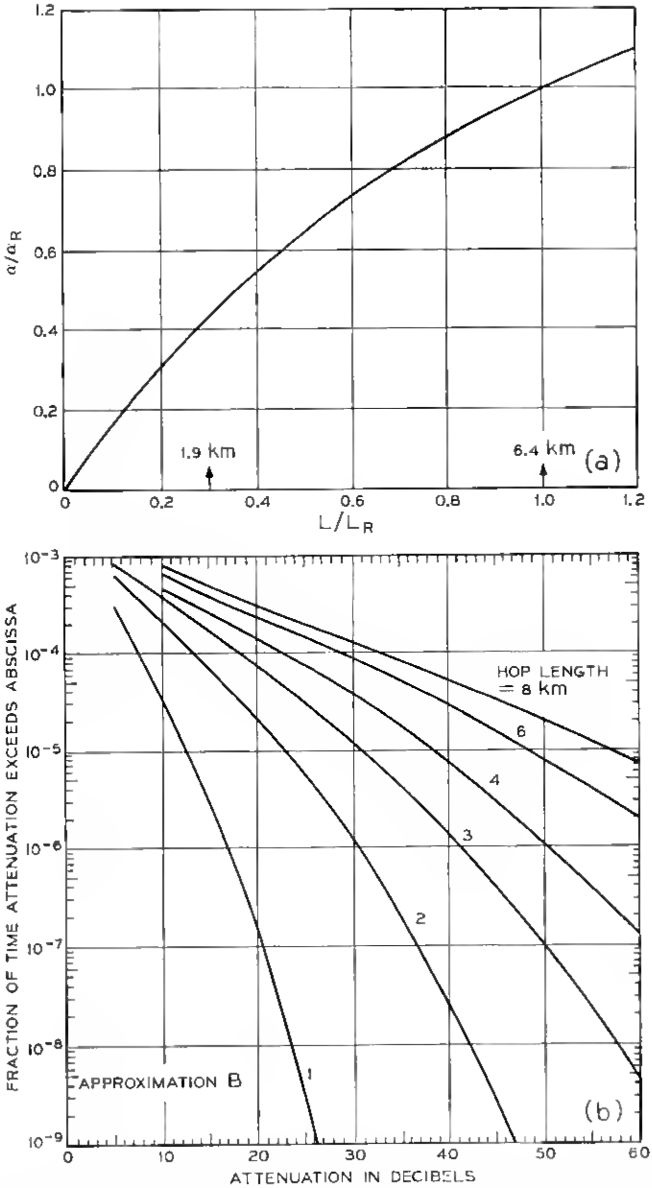

Fig. 3—(a) Attenuation as a function of hop length at constant probability using equation (2) with $\rho = 0.543$. (b) Probability distributions of rain attenuation with hop length as a parameter using approximating function B and equation (2) with $\rho = 0.543$.

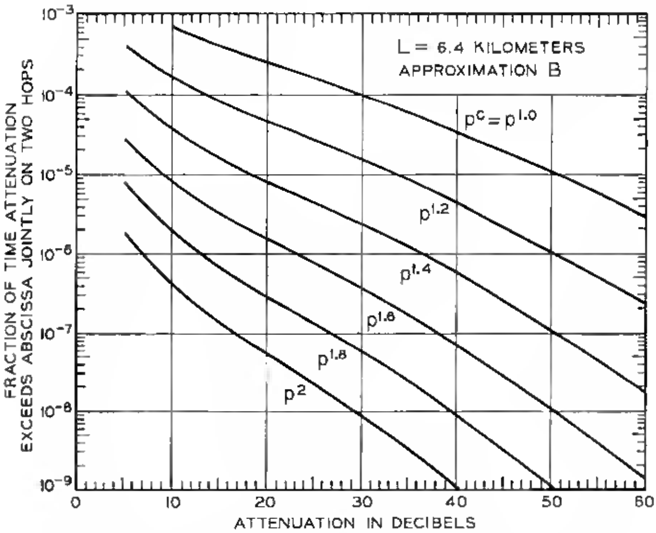

Fig. 4—Joint probability distributions for two paths with c as a parameter using approximating function B and a hop length of 6.4 km.

performance if it is to remain advantageous under more realistic conditions.

The results of this analysis are shown in Figs. 5 through 8. In Figs. 5, 6 and 8, the lengths of tandem and diversity systems with 0.01 percent total rain outage times are plotted as a function of tandem hop length with the joint fading exponent, c, as a parameter of the diversity system. Figure 7 shows the probability that the rain attenuation will exceed the hop fade margin for different hop lengths and fade margins based on approximation B. The results show in general that (*i*) the performance of tandem systems relative to diversity systems increases as the system length increases, making tandem systems more suitable for long systems; (*ii*) the difference in the number of repeaters per unit length required for short and long tandem systems is small; (*iii*) the performance of the diversity system depends strongly on the degree of independence of the two parallel paths in the diversity system; and (*iv*) the performance of the tandem system depends strongly on the tail of the attenuation distribution. Neither of the latter two factors has been measured for rain attenua-

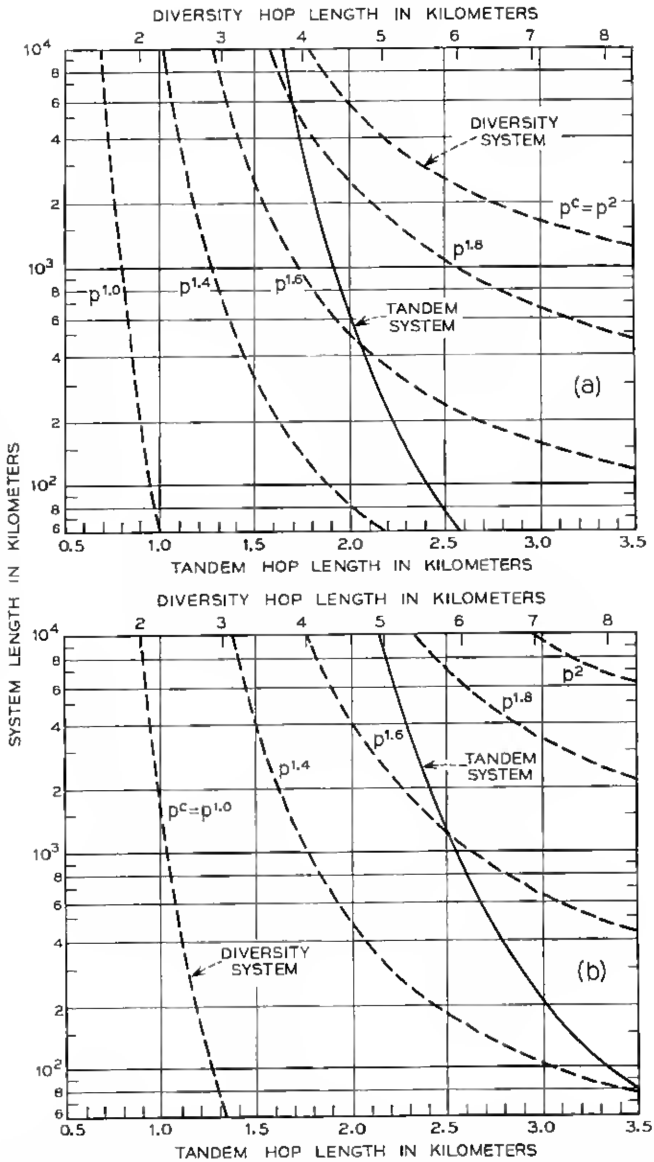

Fig. 5—Lengths of 0.01 percent outage time tandem and diversity systems based on approximating function B with a diversity path separation of 10 km, a section length of 48 km, and a repeater fade margin of (a) 40 dB, and (b) 50 dB.

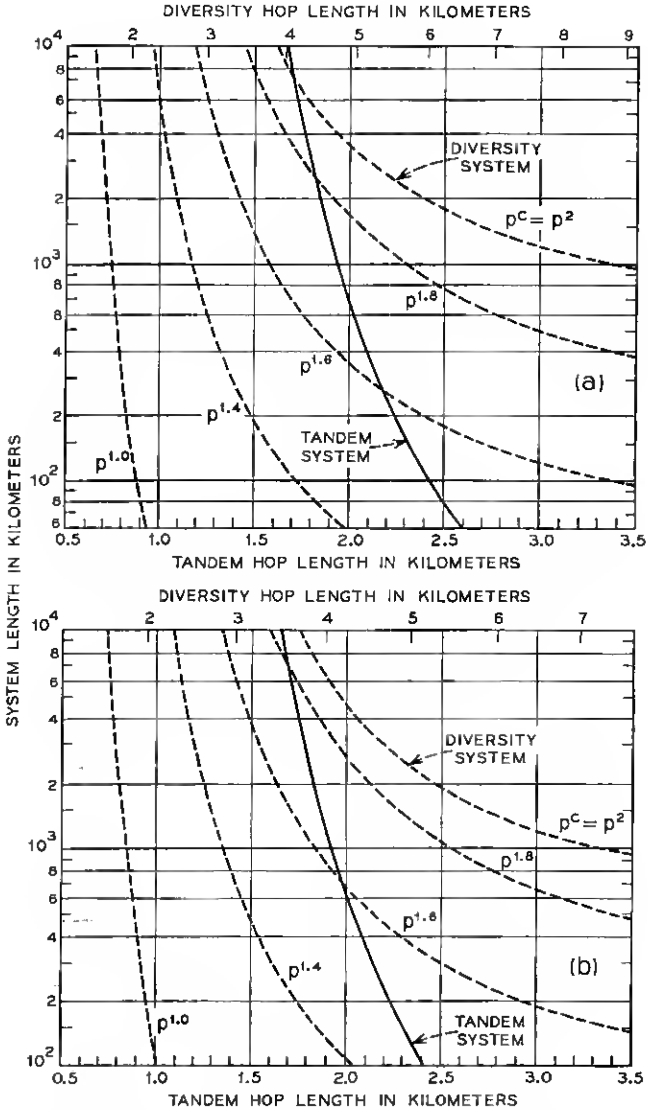

Fig. 6—Lengths of 0.01 percent outage time tandem and diversity systems based on approximating function B with a repeater fade margin of 40 dB, and (a) a 20-km diversity path separation and a 48-km section length, and (b) a 10-km diversity path separation and a 96-km section length.

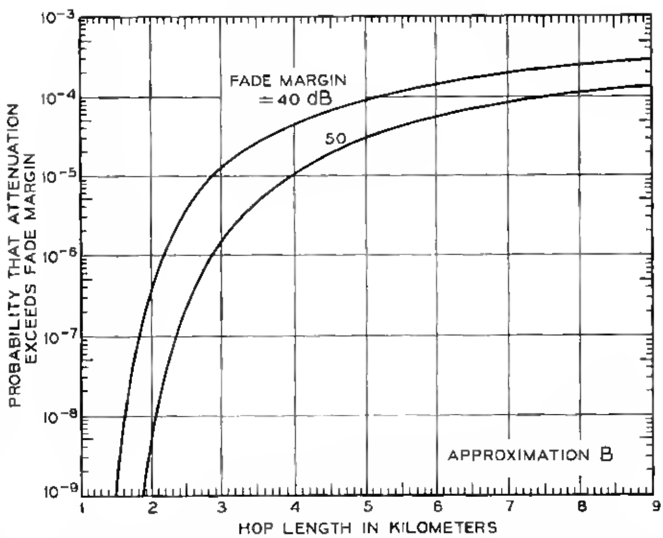

Fig. 7—Probability that the rain attenuation exceeds the hop fade margin as a function of hop length with standard repeater fade margin as a parameter using approximating function B.

tion.* The uncertainty in the tail of the distribution and the sensitivity of the tandem system performance to it results in about a 50 percent uncertainty in the number of repeaters required and emphasizes the need for reliable measurements of the rain attenuation distribution out to a probability of about 10^{-7}. At present, Semplak's measured distribution extends only to about 2×10^{-5}.

III. INTERPRETING THE RESULTS—EXAMPLES

Figure 5(a) shows the system lengths obtained with repeaters having a 40-dB fade margin on a 1-km hop, and with a section length of 48 km and path separation of 10 km. For a tandem hop length of 2 km, the tandem system curve shows that the length of a tandem system with a total rain outage time of 0.01 percent is 650 km. The $c = 2$ diversity curve shows that the length of a diversity system

* The joint *rain rate* distribution reported by Mrs. A. E. Freeny and J. D. Gabbe[7] for a hop length of 5.2 km and path separation of 10.4 km corresponds to $c \approx 1.4$. If it is assumed that the joint attenuation distributions have the same degree of independence, then the performance of the diversity system is indicated by the $c = 1.4$ curves in Figs. 5 through 8 and the diversity system is inferior or comparable to the tandem system depending on the tail of the attenuation distribution.

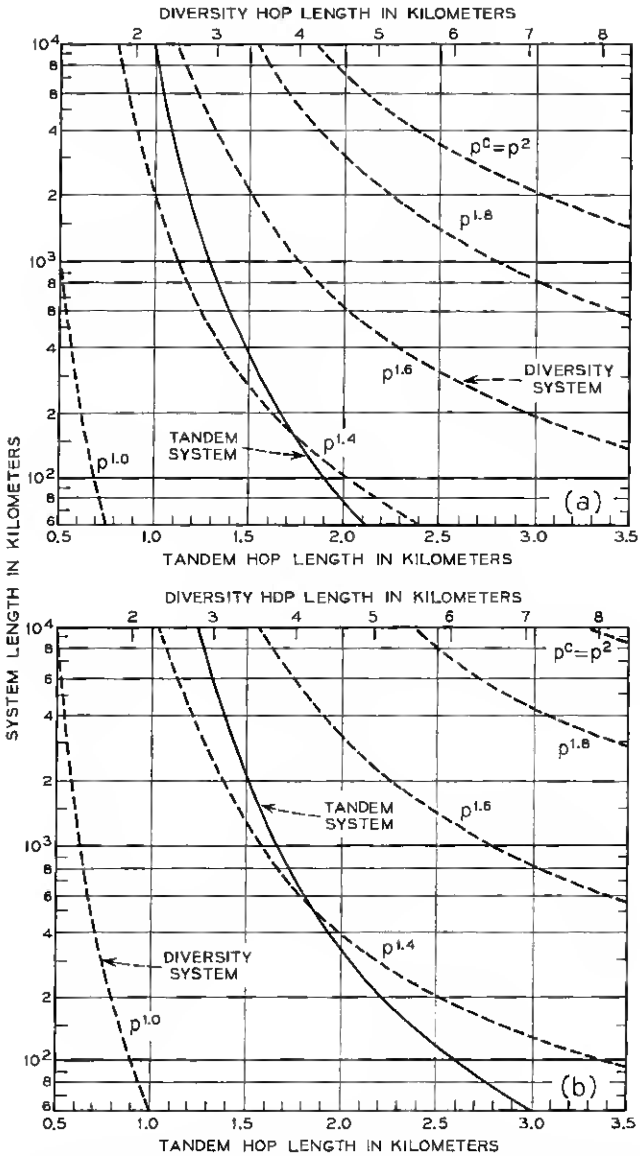

Fig. 8—Lengths of 0.01 percent outage time tandem and diversity systems based on approximating function A with a diversity path separation of 10 km, a section length of 48 km, and a repeater fade margin of (a) 40 dB, and (b) 50 dB.

with the same number of repeaters per units length and the same total rain outage time is 5600 km if all hops have statistically independent fading. However, if there is joint fading of opposite hops corresponding to $c = 1.8$, 1.6, and 1.4, the length of the diversity system decreases to 2450, 510, and 82 km respectively. For joint fading corresponding to $c = 1.6$ the diversity system is better than the tandem system only for system lengths less than 460 km and for $c = 1.4$ the tandem system is better for all lengths.

Figure 5(a) can be interpreted in another way. Suppose a 5000-km system with 0.01 percent rain outage time is required. The tandem system requires a hop length of 1.73 km. Assuming joint fading corresponding to $c = 1.6$, the diversity system requires an equivalent tandem hop length of 1.37 km which is equivalent to a 26 percent increase in the number of repeaters per unit length.

The tandem curve shows another important result. A 500-km short-haul tandem system with 0.01 percent outage time requires a 2.04-km hop length whereas a 5000-km long-haul tandem system with the same outage time requires a 1.73-km hop length. This means that a 10-fold increase in system length requires only an 18 percent increase in the number of repeaters per unit length.

Figures 5(b), 6(a) and 6(b) show the same sets of curves but with different values of the three parameters, repeater fade margin, diversity path separation, and section length. Figure 5 shows that the relative diversity system performance increases with larger fade margins, but the improvement is not appreciable. Figure 6(a) in comparison with Fig. 5(a) can be used to estimate the improvement in joint fading required to offset the larger number of repeaters caused by a wider path separation. Figure 6(b) shows that the relative diversity performance is slightly better for longer section lengths.

The sensitivity of these results to the tail of the attenuation distribution can be seen by comparing Figs. 5 and 8. Figure 8 shows system lengths calculated from approximation A in Fig. 2; otherwise all parameters are the same as in Fig. 5. Because the tandem hops require much smaller fading probabilities than do diversity hops, tandem system performance is very sensitive to the tail of the distribution whereas diversity system performance is fairly insensitive. For example, from Fig. 5(a), using approximation B, a 1000-km tandem system requires a hop length of 1.93 km whereas, from Fig. 8(a) using approximation A, a 1000-km tandem system requires a hop length of 1.29 km, equivalent to a 50 percent increase in the number

of repeaters per unit length. For $c = 1.6$ diversity systems there is no significant difference in hop lengths, reflecting the equality of the two approximations in the region where measured statistics are available.

IV. PROBABILITY DISTRIBUTIONS FOR RAIN ATTENUATION

The rain outage performance of tandem and diversity radio systems depends entirely on the probability distribution of rain induced attenuation on a path of given length and on the joint distributions for multiple paths of different lengths and separations. Such distributions can be obtained in two ways: by direct measurement of attenuation data or by calculation from measured rain rate data. The latter method is much more difficult and unreliable, but one is irresistibly tempted to use it because of the large amount of potentially useful rainfall data available. Consequently, much effort has been spent trying to establish a usable relation between rain rate and attenuation, and, anticipating a usable relation, measurements have been made of the spatial and temporal characteristics of rainfall; but few statistics are available for directly measured rain attenuation. In order to establish the basis for the attenuation distributions used here, these measurements will be discussed briefly.

In 1965, R. G. Medhurst[8] summarized previous measurements and compared measured attenuation with the attenuation computed from measured rain rates using the Mie theory; he found the relationship between them quantitatively inconsistent. Semplak and R. H. Turrin[9] have reported attenuation measurements at 18 GHz on a 6.4-km path in New Jersey and, for data accumulated in 1967, have compared these measurements with attenuations calculated from measured rain rates along the path on a single event basis and also by comparing the annual distribution of measured attenuation with the annual attenuation distribution computed from the annual rain rate distribution. On the single event basis the results were similar to Medhurst's findings. The comparison of the probability distributions showed reasonable agreement if the attenuation was computed from the rain rate by the formula

$$\alpha = k\bar{R}d \tag{1}$$

where k was 0.059, $\bar{R}$ the average rain rate, and d the hop length. However, Semplak[1] has also reported a similar comparison of the distributions based on 1968 data in which case agreement was obtained

with $k = 0.09$. A value of $k = 0.1$ has also been used.[10] Tbese variations translate directly into at least a 50 percent uncertainty in the number of repeaters required if rain rate statistics are used for system calculations.

Freeny and Gabbe[7] have published a statistical summary of rain rate measurements made in a six-month period in 1967 on the Crawford Hill, New Jersey, rain gauge network. This work was aimed at obtaining a statistical description of the behavior of point rain rates, the relationship of two rain rates separated in space, and tbe relationship of average rain rates on pairs of paths in various configurations. However, although believing the relative values to be fairly good, the authors do not believe the numerical values should be considered better than an order of magnitude.

Ruthroff[11] has suggested that the discrepancy between measured attenuations and those calculated from rain rate data may lie in the measurement of the rain rates. He has also shown that because of the integrating effect of the radio patb, the time over which the rain rate is integrated plays an important part in obtaining path attenuation from rain rates along the path.[12]

Thus, for the present, it appears tbat neither the statistics of high rain rates nor the quantitative relation between rain rate and attenuation are known accurately enough for rain rate data to be used as a basis for system design. Our recourse is to use the distributions obtained from directly measured attenuations. The problem in this case is that the amount of data is inadequate, in both the long time statistical sense and in the range of values of the parameters involved. Semplak[1] has reported a three-year average annual attenuation distribution for the 18-GHz attenuation on a 6.4-km path in New Jersey, and a two-year average annual distribution for the 30.9-GHz attenuation on a 1.9-km path at the same location, but no distributions for different path lengths or joint distributions for spatially separated paths are available.

With the above factors in mind, the following approach is used here to obtain the required distributions of attenuation for individual and spatially separated paths. The three-year average annual distribution for measured rain attenuation at 18.5 GHz on the 6.4-km path as reported by Semplak[1] is taken as the basic distribution. This distribution is approximated by two functions, called A and B, which represent reasonably high and low values of the tail of the distribution. Semplak's measured 6.4-km distrihution and the approximating functions

A and B are shown in Fig. 2 as curves R1, A1, and B1 respectively. Two functions are used in order to show the effects of variations in the tail of the distribution since the measured distribution does not extend to sufficiently low probabilities.

In order to introduce a hop length dependence for the distributions, the functional dependence on hop length

$$\frac{\alpha}{\alpha_R} = \frac{L/L_R}{\rho + (1 - \rho)L/L_R} \tag{2}$$

is assumed for the attenuation at any given probability level, where α is the attenuation for a hop length L, α_R is the reference attenuation at the reference length L_R which are taken from the measured 6.4-km distribution, and ρ is a suitably chosen constant. To evaluate ρ, an attenuation distribution for a shorter hop length is needed; for this purpose the two-year average annual distribution for measured rain attenuation at 30.9 GHz on the 1.9-km hop is converted to an 18.5-GHz distribution by dividing the attenuation at any given probability level by 2.0.* For $\alpha_R = 40$ dB and $L_R = 6.4$ km from the 6.4-km distribution, and $\alpha = 17.5$ dB and $L = 1.9$ km from the derived 1.9-km distribution, both at a probability level of 3.45×10^{-5}, the value $\rho = 0.543$ is obtained. Equation (2) with $\rho = 0.543$ is plotted in Fig. 3. This variation of α with L is approximately linear for small values of L but decreases for larger L as is intuitively expected. The curve is similar in shape but somewhat more nonlinear than that obtained by Hogg from rain rate data.[10] The derived 1.9-km distribution and the approximating functions A and B for a 1.9-km hop length are shown in Fig. 2 as curves R2, A2, and B2 respectively. For comparison, the distribution calculated from H. E. Bussey's[13] one-minute Washington, D. C., rain rate distribution, using equation (1) with $k = 0.1$, is also shown in Fig. 2 for a hop length of 1.9 km.

For calculating the outage time of a diversity system, joint probability distributions for two parallel laterally separated hops of equal length are needed for different hop lengths and path separations. Since no such directly measured attenuation distributions are available these distributions must be assumed. Fortunately, because of the assumed symmetry of the diversity system, the distributions for two special cases can be derived from the individual distributions. Let event a_i be the fading of the ith diversity hop and event b_i be the

* This method of obtaining a 1.9-km distribution was first suggested to me by Gusler. The factor of 2.0 has been used previously, but should be viewed with caution since it is derived basically from equation (1).

fading of a hop in the opposite path of the diversity system; the joint probability of both events is $P(a_i, b_j)$. Since the hops are identically distributed, i.e., $P(a_i) = P(b_j) \equiv p$, and only the cases when the variables take the same values are considered, in the special cases of complete dependence $[P(a_i \mid b_j) = P(b_j \mid a_i) = 1]$ and statistical independence the joint probabilities are $P(a_i, b_j) = p$ and $P(a_i, b_j) = p^2$ respectively. Joint probabilities between and including these two cases are represented by

$$P(a_i, b_j) \equiv p^c \qquad 1 \leqq c \leqq 2 \tag{3}$$

where $c = 1$ gives the case of complete dependence and $c = 2$ gives the case of statistical independence. The joint probability distributions generated in this way for 6.4-km hop lengths using approximating function B are shown in Fig. 4 for different values of c. It would be possible to assume some functional dependence of c on diversity path separation and then compute diversity performance for different path separations; the approach used here, however, is to compute performance for a range of values for c at two practical values of path separation. The diversity system performance can then be deduced once the relative independence of two hops is measured.

V. SYSTEM PERFORMANCE

5.1 *Assumptions*

(*i*) The same standard repeater is used in both tandem and diversity systems, i.e., transmitter power, antenna gain, and receiver noise figure are the same. The repeater is characterized by its fade margin on a one-km hop.

(*ii*) The number of repeaters in one section is the same in both tandem and diversity systems, or equivalently, the number of repeaters per unit length is the same in both systems.

(*iii*) The end hops in each diversity path are the same length as the hops in the comparison tandem path, but in calculating joint fading probabilities involving these hops they are considered the same as the other diversity hops. In other words, we assume that the end hops are shorter but that their closer spacing just compensates causing their joint fading probability to be the same as the other diversity pairs.

(*iv*) The rain attenuation probability distribution is given by one of the following approximations to Semplak's measured distribution at 18.5 GHz on a 6.4-km hop.

Approximation A:

$$p \equiv p(\alpha \geqq \alpha_R) = \frac{.0134}{\alpha_R} \epsilon^{-0.0567\alpha_R} . \tag{4}$$

Approximation B:

$$p \equiv p(\alpha \geqq \alpha_R) = \frac{.00719}{\alpha_R} \epsilon^{-0.00103\alpha_R^2} , \tag{5}$$

where α_R is the total rain attenuation. These approximations are shown as curves A1 and B1 respectively in Fig. 2.

(*v*) The attenuation at a constant probability level is related to the hop length according to equation (2) with $\rho = 0.543$, $L_R = 6.4$ km, and α_R equal to the attenuation on the 6.4-km hop. If the attenuation α on a hop with length L is known, the probability that that attenuation is exceeded is found by finding the reference hop attenuation α_R for the same probability from equation (2) then substituting α_R into equations (4) or (5).

(*vi*) All hops have statistically independent fading except the directly opposite hops and the first diagonally opposite hops in the two diversity paths; the joint distribution for these pairs of hops is given by equation (3). As shown in the Appendix, the probability of outage for the diversity system is a series of sums and differences of the joint fading probabilities of combinations of two or more hops. In probabilities involving three hops, directly opposite and first diagonally opposite hops are paired and the pair is assumed statistically independent of the third hop. Joint probabilities involving combinations of four or more hops are assumed to be zero with a maximum estimated error of less than 1.4 percent when $c = 1$.

5.2 *Tandem System*

Let L_t be the tandem repeater spacing and R be the number of tandem repeaters in a section with length S as shown in Fig. 1. The number of hops in the tandem system, N_t, is equal to the number of repeaters, i.e., $N_t = R = S/L_t$. The fade margin of each of the tandem hops is

$$\alpha_t = \alpha_s - 20 \log L_t \tag{6}$$

where α_s is the fade margin of the standard repeater on a 1-km hop.

The fractional rain outage time of the tandem section is

$$P_t = 1 - (1 - p_t)^{N_t} \approx N_t p_t \tag{7}$$

where p_t is found by substituting α_t and L_t into equation (2) and the resulting α_R into equations (4) or (5). The outage time of the section determines the length, L_t, of a system with a specified total outage time, T,

$$L_t = \frac{ST}{P_t}. \tag{8}$$

The system lengths of tandem systems with 0.01 percent total rain outage time are plotted in Figs. 5 through 8.

5.3 *Diversity System*

Referring to Fig. 1, in each diversity path the number of diversity hops, N_d, with the diversity hop length, L_d, is

$$N_d = \frac{R-3}{2} = \frac{S/L_t - 3}{2} \tag{9}$$

and

$$L_d = \frac{S + (\sqrt{2} - 1)D - 2L_t}{N_d} \tag{10}$$

where R is the total number of diversity repeaters, and D is the diversity path separation.

We have assumed that the probability of joint fading of the merge hops is the same as the probability of joint fading on the other diversity hops by virtue of the combination of a shorter length and reduced spacing. Therefore in calculating the probability of the diversity system not working, the effective number of diversity hops in a diversity path is

$$N_{de} = N_d + 2 = \frac{S/L_t + 1}{2}. \tag{11}$$

The fade margin for the diversity hops is

$$\alpha_d = \alpha_s - 20 \log L_d . \tag{12}$$

The fractional rain outage time of the diversity section is (see Appendix)

$$\begin{aligned} P_d = {} & [(3N_{de} - 2)p_d^{(c-2)} + (N_{de}^2 - 3N_{de} + 2)]p_d^2 \\ & - [(6N_{de}^2 - 16N_{de} + 12)p_d^{(c-2)} + (N_{de}^3 - 7N_{de}^2 + 16N_{de} - 12)]p_d^3 \end{aligned} \tag{13}$$

where p_d is found by substituting α_d and L_d into equation (2) and the resulting α_R into equations (4) or (5), and where c is the joint fading

exponent defined in Section III. The system length is calculated from equation (8).

Figures 5 through 8 show the lengths of diversity systems with a 0.01 percent total rain outage time plotted as a function of the hop length of the tandem system which has the same number of repeaters per unit length. Curves are shown for five degrees of joint fading between directly opposite and first diagonally opposite hops. The most severe case of joint fading is $c = 1$, the value corresponding to complete dependence of the two paths.

5.4 *Effects of Other Combinations of Joint Fading*

There are two other types of joint fading which affect the relative merit of tandem and diversity systems. As a result of assuming independence in these cases, the diversity-tandem comparison previously given is more favorable to diversity than is actually the case, perhaps substantially so; and secondly the actual outage time calculations for the tandem system are conservative, predicting more outage time than is actually the case.

5.4.1 *Joint Fading of Series Hops*

Joint fading of the hops connected in series occurs on both tandem and diversity paths. In either case, joint fading of series hops in a path decreases the total outage time for the path because more of the individual hop fades occur at the same time. Therefore the assumption of independence of the series bops gives larger outage times than would be calculated with joint fading. However, since the diversity hop lengths are always at least twice as long as the tandem hop lengths, the average distance hetween series diversity hops is always larger than hetween series tandem hops causing less joint fading of the series diversity hops. Therefore the assumption of independence of series hops increases the calculated outage times more for tandem than for diversity.

5.4.2 *Joint Fading of Diagonally Opposite Hops in the Diversity Paths*

In a diversity system witb 4-km bops spaced 10 km apart, the distance from the center of one hop to the center of the first diagonally opposite hop is 10.4 km; for the next diagonally opposite hops the distances are 10.8 km, 11.2 km, 11.7 km, etc. There is little change in distance for the first few diagonally opposite hops. Therefore the joint fading for these diagonally opposite hops is expected to be comparahle in magnitude to the joint fading of the directly opposite hops. Joint

fading of diagonally opposite hops increases the diversity section outage time in the same way as joint fading of the directly opposite hops. Since joint fading of diagonally opposite hops increases the diversity system outage time, and since the joint fading for at least the near diagonally opposite hops is expected to be comparable to the joint fading of directly opposite hops, the results calculated here on the basis of independence are optimistic toward the diversity system.

VI. ACKNOWLEDGMENT

I wish to express my appreciation to V. K. Prabhu and C. L. Ruthroff for helpful discussions during the course of this work.

APPENDIX

Assume two parallel laterally separated diversity paths with N hops in each path. Denote the fading of the ith hop in one path by event a_i and the fading of the ith hop in the other path by event b_i. Denote the outage of the path containing hop a_i by event a, and the outage of the other path by event b. The probability of a diversity section outage is then

$$P_{nw} = 1 - P(\bar{a} \cup \bar{b}) = 1 - P(\bar{a}) - P(\bar{b}) + P(\bar{a} \cap \bar{b}) \qquad (14)$$

where the bar denotes the complementary event. Using the DeMorgan laws, the $P(\bar{a})$ and $P(\bar{b})$ terms can be evaluated in terms of the individual hop probabilities, e.g.;

$$\begin{aligned} P(\bar{a}) &= P(\bar{a}_1 \cap \bar{a}_2 \cap \bar{a}_3 \cdots) = 1 - P(a_1 \cup a_2 \cup a_3 \cdots) \\ &= 1 - [P(a_1) + P(a_2) + P(a_3) + \cdots - P(a_1a_2) - P(a_1a_3) \\ &\qquad - P(a_2a_3) - \cdots + P(a_1a_2a_3) + \cdots]. \end{aligned} \qquad (15)$$

Similarly, for $P(\bar{a} \cap \bar{b})$ we get

$$P(\bar{a} \cap \bar{b}) = 1 - P(a_1 \cup a_2 \cup a_3 \cdots \cup b_1 \cup b_2 \cup b_3 \cdots). \qquad (16)$$

Expanding equation (16) in a series of joint probabilities of individual events and substituting in equation (14) gives P_{nw} in terms of a series of joint fading probabilities of combinations of two to $2N$ hops excluding joint probabilities involving only hops in the same path. For example, if the $P(\cdot)$ notation is omitted and parentheses are used to indicate pairing of directly opposite and first diagonally opposite hops, P_{nw} has the form

$$P_{nw} = (a_1b_1) + (a_1b_2) + a_1b_3 + \cdots + (b_1a_2) + b_1a_3 + \cdots$$
$$- (a_1b_1)a_2 - (a_1b_1)b_2 - \cdots - a_1(a_2b_2)$$
$$- a_1a_2b_3 - \cdots + (a_1b_1)(a_2b_2) + (a_1b_1)a_2a_3 + \cdots$$
$$+ a_1(a_2b_2)a_3 + \cdots + a_1a_2a_3b_5 + \cdots . \tag{17}$$

From equation (3), the paired terms represent probabilities of the form p^c whereas unpaired individual events have probability p; since they are assumed to be statistically independent P_{nw} has the form

$$P_{nw} = B_{21}p^2p^{(c-2)} + B_{20}p^2 - B_{31}p^3p^{(c-2)} - B_{30}p^3 + B_{42}p^4p^{2(c-2)}$$
$$+ B_{41}p^4p^{(c-2)} + B_{40}p^4 - B_{52}p^5p^{2(c-2)} - B_{51}p^5p^{(c-2)} - B_{50}p^5 + \cdots , \tag{18}$$

where the $B_{\ell m}$ coefficients are the number of terms in P_{nw} with the form $p^{\ell}p^{m(c-2)}$. The $B_{\ell m}$ coefficients are a function of the number of hops N in each diversity path and an exact evaluation of P_{nw} from equation (18) would require evaluating each coefficient for every value of N. Although the total number of terms having the same value of ℓ is easily found, the distribution between coefficients with different m is not. Fortunately the higher order terms get small very rapidly for small p and it can be shown that a conservative estimate of the error involved in assuming that terms of fourth order and higher are zero is less than 1.4 percent if $p \leqq 3 \times 10^{-4}$ and $c = 1.0$, the worst cases used here. For the type of pairing assumed here, B_{20}, B_{21}, B_{30}, and B_{31} have been evaluated giving, from equation (18)

$$P_{nw} = [(3N - 2)p^{(c-2)} + (N^2 - 3N + 2)]p^2$$
$$- [(6N^2 - 16N + 12)p^{(c-2)} + (N^3 - 7N^2 + 16N - 12)]p^3 \tag{19}$$

which is the same as equation (13).

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